

Solutions to the first exercise of homework 1

Remember: risk and expected utility

The main ideas are:

- From the point of view of an agent, the world include events (e.g. getting a job; having an accident; winning the lottery) that are more or less desirable. This is expressed by ascribing a number, the utility, to these events.
- Future events are more or less probable. This is expressed by ascribing a probability p_i , with $0 \leq p_i \leq 1$, to each future event i .
- Agents have to take decisions. They often have to choose among several alternatives. They do that by choosing to do what will lead to the happening of the most desired event (taking into account the cost of doing what it takes for the outcome to happen).
- In most cases, an action will not lead to one single outcome for sure. There is some risk. An action will lead to a distribution of probability among outcomes. E.g. If I apply for this job, there is a $1/2$ that I get the job and $1/2$ that I don't get the job. If I don't apply for the job, the probability that I get the job is zero, and the probability that I don't get it is one.
- In these cases, a *rational* agent calculate the *expected* utility of the possible decisions he can make.
- The expected utility of a decision is the sum of the utilities of each outcome multiplied by the probability that the outcome actually occurs (or, in other words, that the event happens).
- Rational agents compare the expected utility of the decisions that they can make and set for the one that lead to the maximum expected utility.

Tamas: the exercise on risk aversion

In this exercise, you had to compare the expected utility of planting wheat with the expected utility of planting corn. The outcome of each action is monetary gain.

Utility for planting wheat:

$$\frac{2}{3}u(\text{€}2400) + \frac{1}{3}u(\text{€}1800)$$

Utility for planting corn:

$$\frac{2}{3}u(\text{€}1200) + \frac{1}{3}u(\text{€}4200)$$

You are told about the utility $u(x)$ that Tamas derives from monetary gain x .

If Tamas is risk neutral The utility of monetary gain is a linear function of the monetary gain.¹ When comparing the expected utility of monetary gains, it is thus convenient to use the function $u(x) = x$.

The first part of the exercise therefore consists in comparing

$$\frac{2}{3}2400 + \frac{1}{3}1800 \quad \text{and} \quad \frac{2}{3}1200 + \frac{1}{3}4200$$

This turns out to be equal.

The expected utility of planting wheat is equal the expected utility of planting corn, so Tamas is indifferent between the two option.

If Tamas is risk averse We cannot any more use this simplification $u(x) = x$. As said in the text of the exercise, saying that Tamas is risk averse is the same thing as saying that the marginal utility he derives from monetary gain is decreasing. In other words, he enjoys getting more, but for a similar increase of monetary gain, he derives more utility when he initially has little than when he initially has a lot. More generally, for all monetary gain $x > 0$ and for all initial states y_1 and y_2 such that $0 < y_1 < y_2$, we have:

$$u(y_1 + x) - u(y_1) > u(y_2 + x) - u(y_2)$$

¹A linear function is a function of the form $f(x) = \alpha \cdot x + \beta$ with $\alpha, \beta \in \mathbb{R}$

We can easily apply this equation to the expected monetary gains² of Tamas .

$$u(1200 + 1200) - u(1200) > u(2400 + 1200) - u(2400)$$

We then add $u(1200 + 1200) - u(1200)$ to each side of this inequality.

$$\begin{aligned} 2[u(1200 + 1200) - u(1200)] &> u(2400 + 1200) - u(2400) + u(1200 + 1200) - u(1200) \\ &> u(3600) - u(1200) \\ &> u(1200 + 2400) - u(1200) \end{aligned} \quad (1)$$

Applying again the definition of risk aversion, we have:

$$\begin{aligned} u(1200 + 2400) - u(1200) &> u(1800 + 2400) - u(1800) \\ &> u(4200) - u(1800) \end{aligned} \quad (2)$$

The inequalities (1) and (2) give:

$$2[u(2400) - u(1200)] > u(4200) - u(1800)$$

which is equivalent to:

$$2u(2400) + u(1800) > u(4200) + 2u(1200)$$

Dividing each side of the inequality by 3, we obtain:

$$\frac{2}{3}u(2400) + \frac{1}{3}u(1800) > \frac{2}{3}u(1200) + \frac{1}{3}u(4200)$$

This is exactly the relation we were looking for. It says that:

expected utility for planting wheat > expected utility for planting corn

Therefore, if Tamas is risk averse, then he will decide to plant wheat. (By the same reasoning, we should be able to assert that if Tamas is risk seeking, then he will prefer to plant corn.)

²for convenience, I did not write “€” in the arguments of the utility function.